

Muscle Fatigue Analysis by Using a Scale Mixture-based Stochastic Model of Surface EMG Signals

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Abstract—This paper presents the estimation and analysis of surface electromyogram (EMG) signals during fatiguing contractions based on a stochastic EMG model. In the model, the probability distribution of EMG signals is assumed to be a mixture of Gaussians with the same mean but different variances, facilitating the representation of the variance distribution of EMG signals. The paper proposes a continuous estimation method for variance distribution parameters using a sliding window, enabling the evaluation of the time-varying stochastic properties of EMG signals. Estimation experiments were conducted on six healthy young adults to analyze changes in EMG variance distribution with the progression of muscle fatigue. The results reveal the linear and nonlinear relationships between muscle fatigue and variance distribution parameters.

I. INTRODUCTION

Muscle fatigue, which is generally defined as a failure to maintain the required or expected force [1], can be evaluated noninvasively by measuring surface electromyogram (EMG) signals. EMG signals are recorded from the skin surface and reflect the internal state of muscles during fatiguing contractions.

The relationship between EMG variables and muscle fatigue has been studied. In particular, amplitude information, such as root mean square (RMS), and frequency information, such as median frequency (MDF), are known to be representative indices to measure muscle fatigue from EMG signals [2]. These indices reflect the progressive changes in various physiological characteristics during fatigue.

Meanwhile, the stochastic property of EMG signals, particularly its Gaussianity, is known to change depending on muscle activities [3]. One study suggested that the shape of the probability distribution for EMG signals changes due to muscle fatigue [4]. However, the time-series relationship between muscle fatigue and the stochastic properties of EMG signals has not been analyzed in detail.

The authors previously proposed a stochastic model of EMG signals based on a scale mixture distribution with the assumption that the EMG variance is a random variable [5]. It was also shown that the EMG variance distribution may reflect underlying features in muscle activity [5]. Therefore, if we could estimate such parameters from EMG signals during fatiguing contractions, it would be possible to quantitatively evaluate the changes in the stochastic properties due to muscle fatigue.

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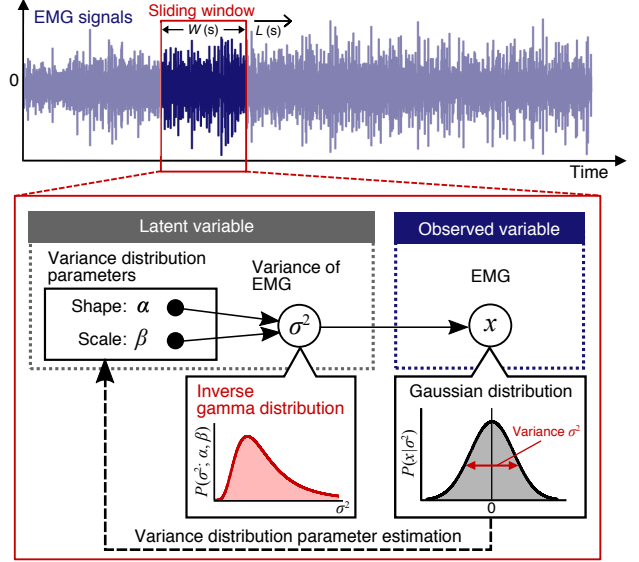


Fig. 1. Overview of the proposed method. The graphical model represents the stochastic relationship between the EMG signal and its variance. The white nodes are random variables, and the black nodes are parameters to be estimated.

This paper analyzes muscle fatigue based on the scale mixture-based stochastic model of EMG signals [5] and highlights variations in the EMG variance distribution during fatiguing contractions. It also outlines an estimation method for variance distribution parameters, by introducing a sliding window, allowing the continuous estimation of time-varying variance distribution parameters from EMG signals.

II. PROPOSED METHOD

Fig. 1 shows an overview of the proposed method. In this method, a time window of length W s is applied to measured EMG signals based on the assumption that the EMG signals are temporally stationary. The signals in the window are based on the scale mixture model, and the model parameters are then estimated. By sliding the window by L s, time-varying stochastic properties of EMG signals can be evaluated.

A. Scale Mixture Model of Surface EMG Signals [5]

The scale mixture model describes the stochastic relationship between the EMG signal x and its variance σ^2 (as illustrated in Fig. 1). In the model, the conditional distribution of an EMG signal x given σ^2 is expressed as a Gaussian distribution with a mean of zero and a variance

of σ^2 :

$$P(x|\sigma^2) = \mathcal{N}(x|\sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{x^2}{2\sigma^2}\right]. \quad (1)$$

The variance σ^2 is also a random variable and is assumed to obey an inverse gamma distribution:

$$P(\sigma^2) = \text{IG}(\sigma^2; \alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} (\sigma^2)^{-\alpha-1} \exp\left[-\frac{\beta}{\sigma^2}\right] \quad (2)$$

where $\alpha \in \mathbb{R}^+$ and $\beta \in \mathbb{R}^+$ determine the inverse gamma distribution and are referred to as the shape and scale, respectively. The scale mixture model of EMG signals is expressed as the marginal distribution of x :

$$P(x) = \int \mathcal{N}(x|\sigma^2) \text{IG}(\sigma^2; \alpha, \beta) d\sigma^2. \quad (3)$$

From this equation, $P(x)$ is obtained by an infinite mixture of Gaussian distributions having the same mean but different variances that are generated by the inverse gamma distribution. The marginal distribution can be expanded as follows:

$$P(x) = \frac{\beta^\alpha \Gamma(\alpha + \frac{1}{2})}{\sqrt{2\pi} \Gamma(\alpha) (\beta + \frac{x^2}{2})^{\alpha + \frac{1}{2}}}. \quad (4)$$

Consequently, the variance σ^2 can be integrated out, and the scale mixture model is determined by the two variance distribution parameters α and β . It should be noted that α reflects the Gaussianity of the EMG signal distribution, and the distribution becomes closer to a Gaussian form as α increases. β reflects the scale of the EMG signals weighted with the Gaussianity [5]. The next subsection outlines a method for the continuous estimation of α and β from measured EMG signals using a sliding window.

B. Continuous Estimation of Variance Distribution Parameters Using a Sliding Window

To evaluate time-varying stochastic properties, the variance distribution parameters are estimated continuously. The data for estimating α and β are obtained from the EMG signals using a sliding window of length W s. Given N samples of data $X = \{x_n\}_{n=1}^N$ obtained from each window, the parameters where observations are most likely to occur can be estimated by maximizing the marginal likelihood $P(X) = \prod_{n=1}^N P(x_n)$. However, the direct optimization of marginal likelihood is generally problematic; hence we conduct this optimization based on the expectation maximization (EM) algorithm [5]. The EM algorithm finds marginal maximum likelihood solutions indirectly by iteratively applying an expectation step (E-step) and a maximization step (M-step).

To simplify the treatment, we first transform (3) via the introduction of the new parameterization $\nu = 2\alpha$ and $s = \beta/\alpha$, with the latent variable redefined as $\tau_n = \sigma_n^2 \alpha / \beta$. Marginal distribution can then be expressed as

$$\begin{aligned} P(x_n) &= \int \mathcal{N}(x_n | s\tau_n) \text{IG}(\tau_n; \nu/2, \nu/2) d\tau_n \\ &= \frac{\Gamma(\frac{\nu+1}{2}) s^{-\frac{1}{2}}}{\sqrt{\pi\nu} \Gamma(\frac{\nu}{2}) (1 + \frac{x_n^2}{\nu s})^{\frac{\nu+1}{2}}}. \end{aligned} \quad (5)$$

The parameters ν and s are estimated as outlined below, by maximizing the marginal likelihood based on the EM algorithm [5].

- (i) Initialize ν and s with the selection of arbitrary starting values.
- (ii) Obtain N samples of data from the current window.
- (iii) E-step. Calculate the expectation of the reciprocal of the latent variable.

$$\omega_n := \mathbb{E}[\tau_n^{-1} | x_n] = \frac{\nu + 1}{\nu + x_n^2/s}. \quad (6)$$

- (iv) M-step. Update s using ω_n .

$$s^{\text{new}} = \frac{1}{N} \sum_{n=1}^N \omega_n x_n^2. \quad (7)$$

Update ν by directly maximizing the log-marginal likelihood numerically based on a line search.

$$\nu^{\text{new}} = \arg \max_{\nu} \ln P(X; \nu, s^{\text{new}}). \quad (8)$$

This is because there is no closed-form expression for ν .

- (v) Evaluate the log-marginal likelihood and check for convergence of it. If convergence is not reached, return to step (iii).
- (vi) Slide the time window by L s and return to step (ii), setting the latest estimation results to be the starting values of ν and s .

Repeat steps (ii)–(vi) until the sliding window reaches the end of the measured EMG signals.

From the above, we can derive the time series estimation results of ν and s . Such results are transformed to variance distribution parameters by $\alpha = \nu/2$ and $\beta = \nu s/2$. Using this procedure, the variance distribution can be estimated continuously.

III. EXPERIMENTS

To analyze the EMG variance distribution observed with the progression of muscle fatigue, estimation experiments were conducted. Six healthy young adults (males, right-handed, age range: 22–25 years) were recruited. Experiments were conducted in accordance with the Declaration of Helsinki. Informed consent was obtained from all study participants before the experiments were performed, and the study was approved by the Hiroshima University Ethics Committee (Registration number: E-840).

During the experiments, participants were seated with their right upper arm pointing downward, the right shoulder abducted at 45° , and the elbow flexed at 90° . The right forearm was pronated, and the fingers, except for the index finger, were fixed. The abduction force produced by the contraction of the first dorsal interosseous (FDI) muscle was measured using a force sensor (Leptrino, CFS018CA101U; 16-bit A/D; sampling frequency: 1,200 Hz) installed on the left side of the index finger. EMG signals were recorded from electrodes attached on the skin surface over the FDI muscle

using a wireless measurement system (Delsys, Trigno; 16-bit A/D; sampling frequency: 2,000 Hz; bandwidth: 20–450 Hz).

Before performing the fatigue task, participants performed at least two maximum voluntary contraction (MVC) trials. Additional MVC trials were performed if the difference in the peak force of the two MVCs was $> 5\%$. There was a rest period at least 2 min between MVC trials. The largest peak force of all conducted trials was taken as the MVC force. In the fatigue task, a target force level corresponding to 30 %MVC was displayed as a horizontal line on a monitor in front of the participants. The participants were also presented with the exerted force in the form of a vertical bar graph in real time. The participants were instructed to exert constant isometric force at 30 %MVC and maintain this force level until exhaustion. The failure point was defined as the time when the average value of the exerted force for the past 1 s deviated from the target force level by 5%. The time from the start of contraction until the failure point was defined as the endurance time.

The variance distribution parameters $[\alpha, \beta]$ were estimated from the measured EMG signals using the sliding window. The length of the window and the sliding width were set to $W = 5$ s and $L = 0.1$ s, respectively. For comparison, RMS and MDF, which are common indices of muscle fatigue, were also calculated from the measured EMG signals using the same sliding window as the variance distribution estimation.

IV. RESULTS

The mean ($\pm$ SD) endurance time was 188.2 ± 34.7 s (range 137–226 s). Figs. 2(a) and (b) show examples of the whole time-series waveforms of the measured EMG signal and force. The red vertical line indicates the failure point of this participant. Fig. 2(c) shows the corresponding results of the variance distribution parameters, RMS, and MDF. Each index was calculated out to the endurance time and was normalized by its initial value.

Fig. 3(a) shows examples of measured EMG signals at 5% and 95% of the endurance time. Density histograms corresponding to these EMG signals are shown in Fig. 3(b). Fitted distributions with the scale mixture model are also shown as solid red lines.

Fig. 4 summarizes the mean of the calculated indices for all participants. The mean value of each index was calculated via linear interpolation because the obtained time points were different between participants. The vertical and horizontal axes, respectively, indicate the normalized value of each index, based on its initial value, and the normalized time, based on each endurance time. The horizontal dashed lines represent the initial values of the indices, i.e., 100% of the initial values.

V. DISCUSSION

In Fig. 2, the participant successfully maintained the target force level until 137 s. The changes in each index during fatiguing contractions can be seen in Fig. 2(c); these include

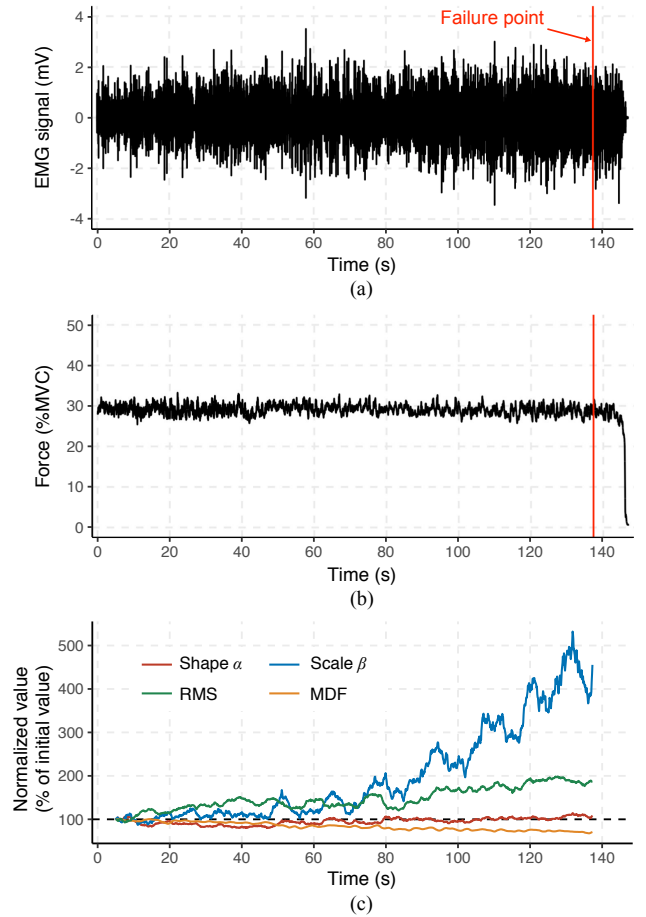


Fig. 2. Examples of measured signals and calculated indices. (a) Measured EMG signal. (b) Measured force. The red vertical line shows the failure point. (c) Calculated indices. Each index was normalized by its initial value.

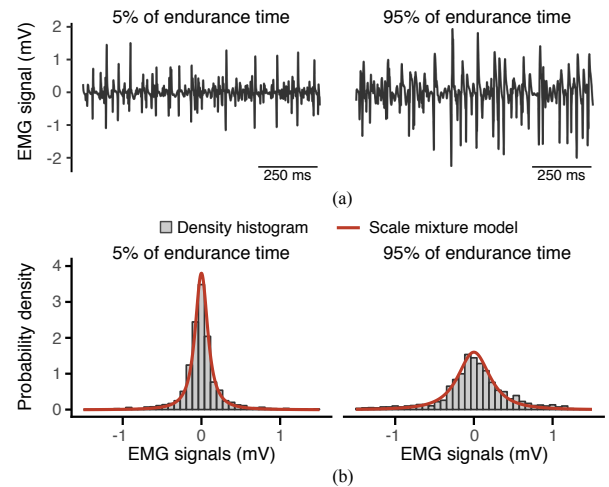


Fig. 3. Examples of measured EMG signals and corresponding density histograms. (a) EMG signals measured at 5% and 95% of the endurance time. (b) Density histograms and fitted distributions based on the scale mixture model at 5% and 95% of the endurance time.

an increase in RMS, a decrease in MDF, a slight increase in α , and an exponential increase in β .

The increase in the EMG amplitude (i.e., RMS) with the

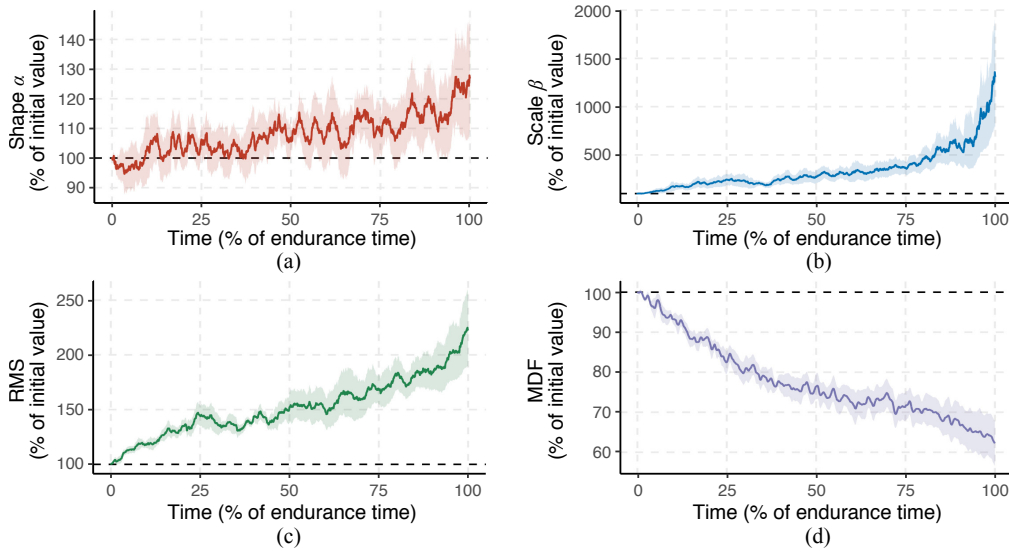


Fig. 4. Mean of calculated indices. (a) Variance distribution parameter α . (b) Variance distribution parameter β . (c) RMS. (d) MDF. Each index was normalized by its initial value. Solid lines and shaded areas represent the mean values and the standard errors, respectively, for all participants.

endurance time is demonstrated in Fig. 3(a). Moreover, the shapes of the density histograms of EMG signals are different at 5% and 95% of the endurance time (Fig. 3(b)). The scale mixture model successfully reflects such distribution characteristics of EMG signals and produces a good fit. This is because the model enables description of changes in the non-Gaussianity of EMG signals associated with muscle activity [5].

In Fig. 4, each index changes with the endurance time throughout the contraction, and its tendency agrees with Fig. 2(c). The RMS and MDF showed linear increase and decrease, respectively. Although both variance distribution parameters increased with the endurance time, only β showed an intense and nonlinear increase near the end of the endurance.

The increase in RMS during fatigue is caused by many factors such as the increases in motor unit firing rate, motor unit synchronization, and additional motor unit recruitment. The decrease in MDF reflects the frequency compression of EMG signals, mainly due to the reduction of muscle fiber conduction velocity associated with muscle fatigue. Such changes of RMS and MDF are in agreement with previous studies [2], [6].

The linear increase in α indicates that the EMG signal distribution gradually becomes closer to a Gaussian distribution during fatiguing contraction. Furui *et al.* analyzed the relationship between the variance distribution parameters and muscle force, suggesting that α is related to motor unit recruitment [5]. Therefore, such changes in α may reflect the increased number of recruited motor units with progressive muscle fatigue. By contrast, β showed a nonlinear increase. It is hard to interpret the meaning of β , because it represents the scale (i.e. amplitude) of EMG signals weighted with the Gaussianity. However, an intense increase in β occurred close to the end of the contraction (after 90% of the endurance time); therefore it may reflect nonlinear physiological pro-

cesses behind progressive muscle fatigue.

VI. CONCLUSION

In this paper, we proposed an evaluation method for muscle fatigue that is based on the stochastic model of surface EMG signals. In the model, the EMG variance is handled as a random variable, and the probability distribution of EMG signals is assumed to be an infinite mixture of Gaussians having the same mean but different variances. We introduced a sliding window to the estimation of the model parameters, enabling the continuous evaluation of the variance distribution.

The experiments revealed the increases in variance distribution parameters involved with muscle fatigue. Although further analysis is necessary for the elucidation of the relationship between the variance distribution and physiological aspects of muscle fatigue, the findings in this paper suggest novel possibilities in quantitative evaluation of muscle fatigue based on the stochastic EMG model. In the future, the authors plan to analyze the relation with muscle fatigue in more detail, by introducing subjective assessment of fatigue and other physiological indices.

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